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# Consumption Taxation and Endogenous Growth in a Model with New Generations\*

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## Abstract

This article studies the implications of consumption taxation on capital accumulation in a one-sector endogenous growth model with finite horizons. A tax on consumption, when tax revenues are lump-sum rebated to consumers, redistributes income between living generations and future, still unborn, generations, and therefore depresses aggregate consumption and raises saving, stimulating capital accumulation and economic growth. If however the resources from taxation are used for financing unproductive public spending, the effect of the consumption tax on the endogenous growth rate disappears as no intergenerational redistribution of income occurs. Finally, a consumption tax hike accompanied by a compensatory reduction of public debt increases long-run economic growth and reduces the consumption-output ratio. Our results on consumption taxation differ substantially from those obtained within the endogenous growth literature.

*Keywords:* Consumption tax; Endogenous growth; Overlapping generations

*JEL classification:* H24; O41

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## 1. Introduction

The developments of the "new growth theory" have stimulated the research investigation into the effects of tax policies on the endogenously determined growth rate of the economy. For example, the contributions of Barro (1990), Rebelo (1991), Barro-Sala-i-Martin (1992), Pecorino (1993), Devereux-Love (1994), Stokey-Rebelo (1995), Turnovsky (1996), Uhlig-Yanagawa (1996), Mendoza-Milesi Ferretti-Asea (1997), Milesi Ferretti-Roubini (1998) and Turnovsky (2000), among others, have recently examined this issue.<sup>1</sup>

The current policy debates on the proper tax incentives capable of stimulating economic growth end up in several cases with the proposal of substituting to some extent income tax with consumption tax with the aim of exempting savings from the burden of double taxation and enhancing capital accumulation as well as output growth; see, for example, Bradford (1986) and McClure-Zodrow (1996).<sup>2</sup>

However, according to the endogenous growth literature that has been developed so far, a tax on consumption either does not influence output growth at all or alternatively reduces it.

A consumption tax exerts no effects on the self-propelling economic growth rate when infinite-horizon one-sector growth models with exogenous labour supply are considered.<sup>3</sup> In such contexts this type of tax works like a lump-sum tax. The inability of a consumption tax to affect growth depends on the fact that it does not alter the optimal decision about the allocation of present and future consumption due to the infinite horizon of the representative agent, as it produces no effects whatsoever on the relative price of consumption today versus consumption tomorrow.<sup>4</sup> Note that the growth rate of output remains constant as the consumption tax varies, whatever the use of the tax revenues is.<sup>5</sup>

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<sup>1</sup>An attempt to survey the consequences of different types of taxation within endogenous growth models can be found in Xu (1994). For a general perspective concerning the consequences of fiscal policy on long-run growth, see the survey of Tanzi-Zee (1997).

<sup>2</sup>This proposal, which is not new but has a long tradition in the history of economic thought, was suggested, among the early proponents, by Fisher (1937) and Kaldor (1955).

<sup>3</sup>See, for example, Barro (1990), Rebelo (1990), Barro-Sala-i-Martin (1992), and Xu (1994).

<sup>4</sup>In other terms the return on consumption is not influenced by the tax, because of the Ricardian debt-neutrality.

<sup>5</sup>The above results have also a correspondence within neoclassical growth models. In the exogenous growth approach with inelastic leisure-labour choice, a permanent increase in the consumption tax rate is neutral for the steady state capital intensity (determined by the *modified golden rule*), consumption and the short-run rate of capital accumulation, provided that revenues

Permanent effects of consumption taxation on growth can be found, even though not necessarily, when an endogenous labour-leisure choice is introduced in models with infinitely-lived consumers. In such a case, the tax distorts the consumption-leisure decision. A consumption tax hike gives rise to substitution and income effects, and the net equilibrium consequence on leisure is obtained by weighing the two effects. Devereux-Love (1994) and Milesi Ferretti-Roubini (1998), for example, discover that the tax reduces the balanced growth rate, because of the net increase in leisure (stemming from a substitution effect greater than the income effect).<sup>6</sup> However, Stokey-Rebelo (1995), incorporating some simplifying assumptions, show that the growth effect of consumption taxation disappears, also when the tax revenues are not rebated to consumers, because income and substitution effects cancel themselves out leaving leisure unaffected.

These conclusions, while being rigorously founded, are in some sense counterintuitive, especially when labour supply is inelastic, because cases could exist in which the consumption tax *per se* distorts the consumption-saving decision, crowding out consumption and stimulating saving,<sup>7</sup> when consumers are lump-sum compensated for the expenditure tax, and therefore implying faster capital accumulation and higher rate of output expansion.

This intuition is supported by two pieces of empirical evidence offered by Lewis (1998) and Lewis-Seidman (1998), on the one side, and Mendoza-Milesi Ferretti-Asea (1997), on the other. In the Lewis (1998) and Lewis-Seidman (1998) papers, it is shown by using U.S. data that a conversion of an income tax to a consumption tax can increase aggregate saving even if the propensity to save of each consumer is maintained constant, because of the variation in the propensity to save among consumers. Mendoza-Milesi Ferretti-Asea (1997) find, by using a cross-country panel of 18 OECD countries, that the consumption tax and the investment rate are positively related, whereas consumption taxation is not a statistically-significant determinant of growth, most probably because outliers represent a serious problem

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of the tax are distributed as lump-sum subsidies to consumers. If the revenues of the tax are used to finance unproductive government spending, we register only a steady-state reduction of consumption and again no effects on long-run capital intensity and capital accumulation along the transition path (see Abel-Blanchard, 1983).

<sup>6</sup>When an endogenous labour supply is introduced in neoclassical growth models, a consumption tax, if tax revenues are rebated to consumers as lump-sum transfers, reduces the long-run capital stock and labour by the same amount in percentage terms, implying that along the transition path the effect on the growth rate of output is negative.

<sup>7</sup>Naturally we are assuming that the substitution effect activated by the tax shock outweighs the income effect.

in the growth regressions.

The purpose of this paper is to investigate the consequences of the consumption tax on capital formation and economic development through a simple one-sector endogenous growth model based on the uncertain lifetimes approach of Blanchard (1985).<sup>8</sup> Labour supply is considered inelastic so as to eliminate the intratemporal consumption-leisure distortion brought about by the consumption tax and to allow us to focus solely on the effect of the shock on the relative price of consumption today and tomorrow.

The analysis of consumption taxation has not yet received too much attention within setups based on finite lives. One exception is represented by Auerbach-Kotlikoff (1987), where the consequences of different types of taxes are investigated through a Samuelson-Diamond OLG neoclassical growth framework with an elastic labour supply. In the simulated model they show that, despite the labor-leisure choice distortion introduced by a non-unit price of consumption, the indirect tax spurs investment and temporarily growth.

We discover that in the endogenous growth model with new generations continuously entering the economy, consumption taxation stimulates saving as consumption is crowded out, allowing for more investment and permanently higher output growth rates, when current tax revenues are lump-sum rebated to currently living consumers. The reason for such a result depends on the fact that the consumption tax affects aggregate saving through the intergenerational redistribution of income that is carried out by the distribution of tax revenues. In fact, a consumption tax is not neutral as a higher tax rate implies greater lump-sum transfers distributed by the government to consumers. A redistribution of wealth between young and older generations occurs, leading to higher aggregate saving and capital accumulation as young people save relatively more than older consumers.<sup>9</sup>

If, instead, the resources from taxation were used for financing unproductive public spending, the net effect of the consumption tax on the growth rate would vanish, because according to this public-spending financing scheme, income is not redistributed between living generations and generations that are still unborn.

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<sup>8</sup>Other papers have already plugged Blanchard's approach into endogenous growth frameworks -see, for example, Alogoskoufis-Van der Ploeg (1990), Saint-Paul (1992), Van der Ploeg-Alogoskoufis (1994), Bertola (1996) and Reinhart (1999).

<sup>9</sup>Therefore in our model the positive effect of the consumption tax on saving and economic growth is due to demographic heterogeneity of households, while in the Lewis (1998) and Lewis-Seidman (1998) experiments this positive effect depends on the income heterogeneity among households.

Finally, a consumption tax hike accompanied by a compensatory reduction of public debt increases long-run economic growth and reduces the share of consumption in national income. This experiment generates a transitional adjustment of the economy, characterized by a moderate short-run variability of output growth and consumption to output ratio.

## 2. The model

Consider a real economy populated by finitely-lived consumers, competitive firms and the government. Time is continuous and agents are endowed with perfect foresight.

The demand-side of the economy is modelled according to the overlapping-generations model of Blanchard (1985) without an operative intergenerational bequest motive. Individuals face uncertainty on the duration of their lives, since they face, when they are alive, a constant probability of death  $\lambda$ . In every instant of time, a large cohort is born. The size of each cohort is normalized to one. Population, composed of the cohorts of all ages, remains constant, since the birth rate is assumed to equal the death rate.<sup>10</sup> Labour is supplied inelastically and hence normalized to one.

We assume that the instantaneous utility function of consumers is logarithmic. The consumer born at time  $s$  maximizes the expected lifetime welfare

$$\int_t^\infty \ln \tilde{c}(s, v) e^{(\theta+\lambda)(t-v)} dv \quad (1)$$

subject to the individual flow budget constraint

$$\frac{d\tilde{f}(s, t)}{dt} = [r(t) + \lambda] \tilde{f}(s, t) + \tilde{y}(s, t) - \tilde{q}(s, t) - (1 + \tau_c) \tilde{c}(s, t) \quad (2)$$

where  $\tilde{c}(s, t)$ ,  $\tilde{f}(s, t)$ ,  $\tilde{y}(s, t)$ ,  $\tilde{q}(s, t)$  denote at time  $t$  consumption, non-human wealth, non-interest income and lump-sum taxes of a consumer born at time  $s \leq t$ , respectively;  $r(t)$  denote the interest rate at time  $t$ ;  $\theta$  and  $\tau_c$  are the exogenous rate of time preference and the constant consumption tax rate, respectively. Tildes denote individual variables.

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<sup>10</sup>A demographic structure based on overlapping infinitely-lived families not altruistically linked to older cohorts that enter the economy continuously (as in Weil, 1989) would not affect our results.

The budget constraint (2) incorporates the hypothesis that consumers receive an actuarially fair premium  $\lambda \tilde{f}(s, t)$  from the life insurance company and give all their wealth to the life insurance company contingent on their death.

By integrating the budget constraint (2) forward and by imposing the transversality condition precluding Ponzi games, we obtain the consumer's intertemporal budget constraint

$$\int_t^\infty \tilde{c}(s, v) e^{-\int_t^v [r(u) + \lambda] du} dv = \frac{[\tilde{f}(s, t) + \tilde{h}(s, t)]}{(1 + \tau_c)} \quad (3)$$

where  $\tilde{h}(s, t)$  represents human wealth, i.e. the present discounted value of expected future income net of lump-sum taxes.

The first order conditions for the individual maximization problem yield

$$\frac{d \tilde{c}(s, t)}{dt} = [r(t) - \theta] \tilde{c}(s, t) \quad (4')$$

and thus from the consumer's present value budget constraint

$$\tilde{c}(s, v) = \frac{(\theta + \lambda)}{(1 + \tau_c)} [\tilde{f}(s, t) + \tilde{h}(s, t)] \quad (4'')$$

Note that the introduction of a proportional tax on consumption decreases the marginal propensity to consume out of wealth, because total wealth should be deflated, in order to be expressed in terms of consumption, by the price of one unit consumption, given by  $(1 + \tau_c)$ .

After having aggregated the solution of the individual maximization program over all the cohorts,<sup>11</sup> we obtain the Blanchard-Yaari law of motion of consumption, and the dynamic equations of aggregate non-human and human wealth

$$\dot{C}(t) = [r(t) - \theta]C(t) - \frac{\lambda(\theta + \lambda)}{(1 + \tau_c)}F(t) \quad (5a)$$

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<sup>11</sup>Each aggregate variable is defined as

$$Z(t) = \int_{-\infty}^t \tilde{z}(s, t) \lambda e^{\lambda(s-t)} ds$$

where  $\tilde{z}(s, t)$  indicates a generic individual variable. Therefore capital letters denote aggregate variables of the corresponding lower-case letters with tilde.

$$\dot{F}(t) = r(t)F(t) + Y(t) - Q(t) - (1 + \tau_c)C(t) \quad (5b)$$

$$\dot{H}(t) = [r(t) + \lambda]H(t) - Y(t) + Q(t) \quad (5c)$$

Since the rate of change of aggregate consumption depends, amongst other factors, on the level of non-human wealth, it is also affected by the consumption tax rate, which decreases one unit of wealth by  $\frac{\tau_c}{(1 + \tau_c)}$ , because of the non-unit price of consumption.<sup>12</sup>

Non-human wealth of consumers consists of physical capital and stock of government debt

$$F(t) = K(t) + B(t) \quad (6)$$

where  $K(t)$  and  $B(t)$  denote the capital stock and the stock of public debt, respectively.

The production side of the economy is composed of many identical competitive firms. Firms produce output by using the following production function

$$Y(t) = AK(t)^\alpha L(t)^{1-\alpha} K^*(t)^{1-\alpha}, \quad 0 < \alpha < 1 \quad (7)$$

where  $A$  is a positive constant,  $L(t)$  is labour and  $K^*(t)$  is the average capital stock of the economy.  $K^*(t)$  represents a factor of production externality given to the individual firm, due to a "learning by doing" mechanism (as in Romer, 1986). Technology (7) guarantees the existence of an endogenously driven growth rate of output, since it shows constant returns to scale with respect to aggregate capital stock, i.e. the accumulating factor.

The representative firm solves the following intertemporal optimization problem<sup>13</sup>

$$\max_{K(v), L(v)} \Pi(t) = \int_t^\infty \left[ Y(v) - w(v)L(v) - \dot{K}(v) \right] e^{-\int_t^v r(u)du} dv \quad (8)$$

subject to (7).  $\Pi(t)$  represents the present discounted value of net profits.

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<sup>12</sup>The reason for having this effect in the dynamic equation of aggregate consumption is due to the different rates of return on human and non-human wealth. In the case of an infinite horizon ( $\lambda = 0$ ), the effect of the consumption tax rate disappears from equation (5).

<sup>13</sup>For the sake of simplicity, we assume that capital does not depreciate and all capital accumulation (decumulation) occurs at a continuous rate and does not incur adjustment costs.

The first order conditions for the maximum entail

$$r(t) = \alpha AK(t)^{\alpha-1} L(t)^{1-\alpha} K^*(t)^{1-\alpha} \quad (9a)$$

$$w(t) = (1 - \alpha)AK(t)^{\alpha} L(t)^{-\alpha} K^*(t)^{1-\alpha} \quad (9b)$$

Equation (9a) implies that -once the equilibrium condition on the labour market, i.e.  $L(t) = 1$ , and the consistency requirement about  $K^*$ , i.e.  $K^*(t) = K(t)$ , are accounted for- the return on investment is constant, since we have

$$r(t) = \alpha A \quad (9a')$$

This is the key element for having endogenous growth.

Then, we have to consider the government instantaneous budget constraint given by

$$Q(t) + \tau_c C(t) + \dot{B}(t) = G(t) + r(t)B(t) \quad (10)$$

where  $G(t)$  represents government consumption. Equation (10) says that total public outlays, government spending plus interest on public debt are financed through lump-sum taxes, taxes on consumption and the issuance of new debt.

The solvency of the government is guaranteed by imposing the following transversality condition

$$\lim_{v \rightarrow \infty} B(v) e^{-\int_t^v r(u) du} = 0 \quad (11)$$

Finally, the resources constraint of the economy must be considered

$$Y(t) = C(t) + G(t) + \dot{K}(t) \quad (12)$$

Equation (12) states that the full employment output must be equal to the aggregate demand, given by consumption, public spending and private investment, i.e. capital accumulation.

The full macroeconomic equilibrium for the economy is obtained by combining the optimal conditions for consumers and firms, together with the relevant accumulation equations. For simplicity, we can divide all variables by national product and denote them by lower-case letters. After substituting out the capital-output ratio and the interest rate, and dropping the time index, then the model can be expressed in compact form as

$$\dot{c} = (\alpha A - \theta - \gamma)c - \frac{\lambda(\theta + \lambda)}{(1 + \tau_c)}(A^{-1} + b) \quad (13a)$$

$$c = 1 - g - \frac{\gamma}{A} \quad (13b)$$

$$\dot{b} = (\alpha A - \gamma)b + g - q - \tau_c c \quad (13c)$$

where  $\gamma$  is the growth rate of output (equal to the growth rate of capital).<sup>14</sup>

We assume that  $\alpha A > \theta$ , i.e. the economy is considered to be dynamically efficient.

Note that, when no consumption externalities due to finite horizons and no operative bequests hypotheses (i.e.  $\lambda = 0$ ) exist, the model reduces to the infinite-horizon representative-consumer economy and the consumption tax drops out of the system (13).

### 3. Effects of consumption taxation

Our analysis will consider the macroeconomic effects of the following consumption tax experiments: i) an increase in  $\tau_c$  when tax revenues are lump-sum rebated to consumers; ii) an increase in  $\tau_c$  when resources from consumption taxation are used for financing public spending; iii) an increase in  $\tau_c$  when the revenues collected by consumption taxation are employed to reduce public debt.

i) *increase in  $\tau_c$  accompanied by a compensatory reduction in  $q$*

In the present experiment, public debt to output ratio and government expenditure to output ratio remain constant at level  $b = \hat{b}$  and  $g = \hat{g}$ , respectively. This implies that  $q$  is solved residually from equation (13c).

After using equation (13b) to eliminate  $c$  and  $\dot{c}$  (by taking the time derivative) from equation (13a), we obtain the following nonlinear differential equation with constant coefficients

$$\dot{\gamma} = -\gamma^2 + \Omega\gamma + \left[ \Phi(A\hat{b} + 1) - A(1 - \hat{g})(\alpha A - \theta) \right] \quad (14)$$

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<sup>14</sup>Equation (11) -expressed in per output terms- must also be included in the compact model (13):

$$\lim_{v \rightarrow \infty} b(v) e^{-\int_t^v [\alpha A - \gamma(u)] du} = 0 \quad (13d)$$

where  $\Omega = A(1 - \hat{g}) + (\alpha A - \theta) > 0$  and  $\Phi = \frac{\lambda(\lambda + \theta)}{(1 + \tau_c)} > 0$ .

The balanced growth solution can be obtained by setting  $\dot{\gamma} = 0$  in (14) and solving the implied quadratic equation. There are two positive solutions for the growth rate, given by

$$\bar{\gamma}_1 = \frac{\Omega - \left\{ [A(1 - \hat{g}) - (\alpha A - \theta)]^2 + 4\Phi(A \hat{b} + 1) \right\}^{\frac{1}{2}}}{2} \quad (15a)$$

and

$$\bar{\gamma}_2 = \bar{\gamma}_1 + \left\{ [A(1 - \hat{g}) - (\alpha A - \theta)]^2 + 4\Phi(A \hat{b} + 1) \right\}^{\frac{1}{2}} \quad (15b)$$

where the overbar denotes the long-run equilibrium growth rate .

It is straightforward to show that  $\bar{\gamma}_1$  is positive,  $\bar{\gamma}_1 < \alpha A - \theta$  and  $\bar{\gamma}_1 < \bar{\gamma}_2$ .

Figure 1 illustrates the phase diagram for the differential equation (14). There are two stationary equilibria corresponding to roots (15a) and (15b). The stationary equilibrium  $\bar{\gamma}_1$  is locally unstable, while the stationary equilibrium  $\bar{\gamma}_2$  is locally stable. The  $\bar{\gamma}_1$  solution represents the unique steady-state solution as  $\gamma$  (namely  $c$ ) is a nonpredetermined variable, i.e.  $\gamma(0)$  is free.<sup>15</sup> Consequently, there are no transitional dynamics in  $\gamma$  (and hence in  $c$ ) as in response to any shock the output growth rate immediately jumps to its new equilibrium value.

INSERT FIGURE 1

Solution (15a) for the growth rate can be easily employed for evaluating the consequence of the consumption tax on the growth rate. The effect of an increase in  $\tau_c$  is given by

$$\frac{d \bar{\gamma}_1}{d \tau_c} = \frac{\left\{ [A(1 - \hat{g}) - (\alpha A - \theta)]^2 + 4\Phi(A \hat{b} + 1) \right\}^{-\frac{1}{2}} \Phi(A \hat{b} + 1)}{(1 + \tau_c)} > 0$$

An increase in the consumption tax raises the endogenous growth rate of output unambiguously and reduces the consumption-output ratio, as  $\frac{d \bar{c}_1}{d \tau_c} = -\frac{1}{A} \frac{d \bar{\gamma}_1}{d \tau_c} < 0$ .<sup>16</sup> The economy instantaneously jumps from  $P$  to  $P'$  in Figure

<sup>15</sup>In addition, note that  $\bar{\gamma}_2$  corresponds to a negative share of consumption over income.

<sup>16</sup>It is not difficult to ascertain that  $\bar{c} (1 + \tau_c)$  increases.

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The motivation for such a result is to be found in the hypothesis of overlapping-generations with new entries combined with the tax revenue distribution scheme adopted. In this context the consumption tax is capable of permanently affecting saving behaviour. When there are finite lives with new births, a consumption tax affects the saving-consumption decision at aggregate level. Since young people save more than old people, a redistribution of wealth occurs among generations through lump-sum rebates of the tax revenues, in particular between the living generations and the still unborn generations; the current generation receives part of the lump-sum benefits in the form of lower lump-sum taxes. Savings and therefore growth are raised by the tax rate hike.<sup>17</sup> Another way of looking at these effects is the following: the rise in consumption tax brings about a reduction of the return on consumption at aggregate level,<sup>18</sup> inducing people to consume less and save relatively more, therefore leading to more capital accumulation. Hence the intertemporal perfect arbitrage condition between investment and consumption returns is re-established only through an increase in the growth rate.

Note that the effect of consumption tax on economic growth diminishes as long as  $\bar{\gamma}_1$  asymptotically approaches the value of  $\alpha A - \theta$  when  $\lambda$  vanishes. The positive enhancing effect of the consumption tax over the economy's rate of expansion disappears in the limit case of an infinite horizon economy ( $\lambda = 0$  and  $\bar{\gamma}_1 = \alpha A - \theta$ ) -confirming the results of Barro (1990), Rebelo (1990), Barro and Sala-i-Martin (1992), and Xu (1994)- as debt-neutrality holds and the intergenerational redistributive consequences of the tax disappear.

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<sup>17</sup>This result has something in common with Bertola (1996) and Uhlig-Yanagawa (1996), where it is shown, by using a Blanchard-type overlapping generations model and a Diamond-Samuelson type two-overlapping generations setup respectively, that a tax on capital income stimulates long-run growth. The driving force behind their result is identical to the one in our analysis: a tax on capital income redistributes wealth from the old to the young and thus stimulates savings.

<sup>18</sup>The return on aggregate consumption is given by:  $\gamma + \theta + \frac{\lambda(\lambda + \theta)(A \hat{b} + 1)}{(1 + \tau_c)A \bar{c}}$ .

ii) *increase in  $\tau_c$  accompanied by a compensatory increase in  $g$*

In this experiment the government keeps the budget in equilibrium by adjusting the public expenditure to income ratio, rather than  $q$ . By substituting out the government budget constraint for eliminating  $g$  into equation (13b), we obtain

$$(1 + \tau_c)c = (1 - \hat{q} + \alpha A \hat{b}) - \frac{(A \hat{b} + 1)}{A} \gamma \quad (16)$$

After using equation (16) into equation (13a), the following differential equation is obtained

$$\dot{\gamma} = -\gamma^2 + [\Gamma + (\alpha A - \theta)]\gamma + [\lambda(\lambda + \theta) - (\alpha A - \theta)\Gamma] \quad (17)$$

where  $\Gamma = \frac{A(1 - \hat{q} + \alpha A \hat{b})}{(A \hat{b} + 1)} > 0$ .

When  $\dot{\gamma} = 0$ , the unique meaningful solution for the long-run growth rate to the quadratic equation in (17) is

$$\bar{\gamma}_1 = \frac{[\Gamma + (\alpha A - \theta)] - \{[\Gamma - (\alpha A - \theta)]^2 + 4\lambda(\lambda + \theta)\}^{\frac{1}{2}}}{2} \quad (18)$$

According to expression (18), when the government budget is balanced through the endogenous adjustment of public spending to income ratio, long-run economic growth becomes independent of the consumption tax. As the tax revenues are not rebated in a lump-sum fashion but used for government consumption, the above-mentioned redistribution of income among the existing younger generation and other generations is absent and so is the effect on savings as well as on output growth. The increase in  $g$  (induced by the rise in  $\tau_c$ ) fully crowds out private consumption so as to leave capital accumulation,  $\bar{c} + \bar{g}$  and  $\bar{c} (1 + \tau_c)$  unchanged.

iii) *increase in  $\tau_c$  accompanied by a compensatory adjustment of  $b$*

When the government budget is balanced through the adjustment of public debt to output ratio, a tax rule that stabilizes government debt must be considered. We consider the following tax rule

$$q = \varepsilon b \quad (19)$$

where  $\epsilon > 0$ . Equation (19) satisfies the government solvency condition if  $\epsilon > \alpha A - \gamma$ . We assume, without sacrificing the generality of the results, that the stronger condition  $\epsilon > \alpha A$  is satisfied.

The dynamic system can be expressed as the following system of nonlinear differential equations

$$\dot{\gamma} = -\gamma^2 + \Omega\gamma - \left[ A(1 - \hat{g})(\alpha A - \theta) - \Phi(Ab + 1) \right] \quad (20a)$$

$$\dot{b} = \left( \frac{\tau_c}{A} - b \right) \gamma + (\alpha A - \epsilon)b + [\hat{g} - \tau_c(1 - \hat{g})] \quad (20b)$$

where  $\Omega$  and  $\Phi$ , both positive, have been defined before.

The determination of the equilibrium is illustrated in Figure 2. Here, we plot the  $\dot{\gamma} = 0$  schedule, which shows the public debt-output ratio that guarantees a constant growth rate for each value of  $\gamma$ .<sup>19</sup> In Figure 2, we also plot the  $\dot{b} = 0$  locus. It consists of pairs  $(\gamma, b)$  that ensure that the government budget is balanced.<sup>20</sup>

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<sup>19</sup>The equation of the  $\dot{\gamma} = 0$  schedule is given by

$$b = \frac{1}{\Phi A} \left\{ \gamma^2 - \Omega\gamma + [A(1 - \hat{g})(\alpha A - \theta) - \Phi] \right\} \quad (20a')$$

Equation (20a') is quadratic, having a positive intercept on the vertical axis and intersecting the horizontal axis twice, at  $\gamma_1$  and  $\gamma_2$ . These two points of intersection are given by the expressions (15a) and (15b) by setting  $\hat{b} = 0$ . The  $\dot{\gamma} = 0$  locus has a minimum at point  $\gamma^* = \frac{\Omega}{2}$ . The slope of the constant output growth locus could be either positive or negative, depending on whether  $\gamma$  exceeds or falls short of  $\gamma^*$ . Notice that, as for  $\gamma \in (\gamma_1, \gamma_2)$  the corresponding  $b$  is negative and for  $\gamma \geq \gamma_2$  the share of consumption over income is negative, the only section of the  $\dot{\gamma} = 0$  locus that we are interested in is that corresponding to the interval  $[0, \gamma_1]$  for the growth rate.

<sup>20</sup>The equation of the  $\dot{b} = 0$  locus is

$$b = \frac{\tau_c \gamma + A[\hat{g}(1 + \tau_c) - \tau_c]}{A(\epsilon + \gamma - \alpha A)} \quad (20b')$$

This schedule has positive intercept on the vertical axis -as we can reasonably assume  $\hat{g}(1 + \tau_c) > \tau_c$ - and a horizontal asymptote, given by  $b = \frac{\tau_c}{A}$ . The  $\dot{b} = 0$  locus is upward-sloping if  $\epsilon > A[\hat{g} \frac{(1 + \tau_c)}{\tau_c} - (1 - \alpha)]$ . This is the case assumed, without affecting the qualitative results, in Figure 2. If otherwise  $\epsilon < A[\hat{g} \frac{(1 + \tau_c)}{\tau_c} - (1 - \alpha)]$ , the  $\dot{b} = 0$  locus is negatively-sloped.

INSERT FIGURE 2

The steady-state equilibrium is given by the intersection of the two loci at point  $P$ . The equilibrium point  $Q$  can be ruled out as it does not satisfy the stability conditions and corresponds to a negative consumption-output ratio.

The stability of equilibrium in model (20) can be investigated as follows. By linearizing the differential equations (20a) and (20b) around the steady-state equilibrium, we can write the dynamic system in compact matrix form as

$$\begin{bmatrix} \dot{\gamma} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \begin{bmatrix} \gamma - \bar{\gamma} \\ b - \bar{b} \end{bmatrix} \quad (21)$$

where overbar variables denote long-run values and

$$j_{11} = -2\bar{\gamma} + \Omega > 0;^{21}$$

$$j_{12} = \Phi A > 0;$$

$$j_{21} = \frac{\tau_c}{A} - \bar{b};$$

$$j_{22} = -(\varepsilon + \bar{\gamma} - \alpha A) < 0.$$

In order to ensure saddle-point stability, the above Jacobian must have one positive eigenvalue associated with the jump variable,  $\gamma$ , and one negative eigenvalue associated with the predetermined variable,  $b$ . The determinant of the Jacobian, equal to  $|J| = -[(\varepsilon + \bar{\gamma} - \alpha A)(-2\bar{\gamma} + \Omega) + \Phi(\tau_c - A\bar{b})]$ , is negative for reasonable values of structural parameters, as the required condition for saddle-point stability.<sup>22</sup>

The equation of the stable manifold is

$$\gamma = \bar{\gamma} + \Xi(b - \bar{b}) \quad (22)$$

where  $\Xi = \frac{j_{12}}{\eta_1 - j_{11}} = \frac{\eta_1 - j_{22}}{j_{21}} < 0$  and  $\eta_1 < 0$  denotes the stable eigenvalue of the Jacobian in (21). The saddle-path, depicted in Figure 3, is negatively-sloped in the  $\gamma$ - $b$  space.<sup>23</sup>

<sup>21</sup> $j_{11}$  represents the slope of the linearized  $\dot{\gamma} = 0$  locus, taken with a negative sign. Therefore  $j_{11}$  is positive as the meaningful section of the  $\dot{\gamma} = 0$  locus is defined over the interval  $[0, \gamma_1]$  for  $\gamma$ , where this locus slopes downward. See note 19.

<sup>22</sup>Notice that, in the neighborhoods of point  $Q$ ,  $j_{11} < 0$  and hence saddle-point stability is not satisfied.

<sup>23</sup>Notice that the saddle-path is always negatively sloped regardless the sign of  $j_{21}$ . In Figure 3, also the linearized  $\dot{\gamma} = 0$  locus, whose slope is  $-\frac{j_{11}}{j_{12}}$ , and the linearized  $\dot{b} = 0$  locus, whose slope is  $-\frac{j_{21}}{j_{22}}$ , have been drawn. Here the  $\dot{b} = 0$  locus is assumed to be positively sloped to maintain consistency with Figure 2. See also note 20.

### INSERT FIGURE 3

An increase in  $\tau_c$  leads to an upward shift of the whole  $\dot{\gamma}=0$  locus and a counter clockwise rotation of the  $\dot{b}=0$  locus in Figure 2. The equilibrium moves from point  $P$  to point  $P'$ . This implies that higher consumption taxation increases economic growth and reduces both consumption and public debt to income ratios.<sup>24</sup> The intuition behind these results is straightforward. The greater tax revenues stemming from the rise in the consumption tax rate require a reduction of public debt to keep the government budget balanced. Therefore a redistribution of wealth among generations occurs, as government bonds are considered net wealth by current generations, which decreases aggregate consumption. This in turn stimulates national savings, spurring capital accumulation and output growth.

The equilibrium and the associated dynamics are illustrated in Figure 3. As a rise in  $\tau_c$  results in a steady-state increase of the growth rate and a reduction of the ratio of government debt to output, the unexpected permanent fiscal shock shifts the saddle-path upward to  $S'S'$ , leading to an instantaneous jump of economic growth, which undershoots its new long-run equilibrium value.<sup>25</sup> The equilibrium moves suddenly from point  $P$  to point  $P_0$  on the new saddle-path. At  $P_0$  a government budget surplus occurs. Soon after the shock has taken place, the system converges monotonically to the new long-run equilibrium with a decumulation of government debt, a further rise in the growth rate and a reduction of consumption to national income ratio.

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<sup>24</sup>By taking the differentials of equations (20) when  $\dot{\gamma}=\dot{b}=0$ , we obtain the following long-run multipliers:

$$\frac{d\bar{\gamma}}{d\tau_c} = -\frac{[(\varepsilon + \bar{\gamma} - \alpha A)(A\bar{b} + 1)\Phi/(1 + \tau_c) + A\bar{c}]}{|J|} > 0;$$

$$\frac{d\bar{b}}{d\tau_c} = \frac{[(A\bar{b} - \tau_c)(A\bar{b} + 1)\Phi/(1 + \tau_c) + (-2\bar{\gamma} + \Omega)A\bar{c}]}{|J|} < 0;$$

where  $|J| < 0$  has been defined before.

<sup>25</sup>The case of perverse-shooting seems implausible according to plausible structural parameter values.

## 4. Conclusions

In this article we have investigated the consequences of a consumption tax on the economy's rate of output expansion in an OLG endogenous growth setup with new entry.

We have discovered that consumption taxation reduces aggregate consumption and raises saving, stimulating capital accumulation and economic growth, when consumers are lump-sum compensated for the tax. Even if this result is consistent with the intuition, it is quite a new one in the literature that analyzes the effects of taxes adopting one-sector endogenous growth models. The demographic framework and the scheme of distribution of the tax revenues represents the crucial elements for obtaining such a result. In fact our demographic structure fails to display the Ricardian debt-neutrality, so that the intertemporal pattern of net lump-sum taxes to individuals has real effects.

Changes in the consumption tax necessary to finance higher government consumption to output ratio produces no effects on the output growth rate as the intergenerational redistributive effects seen in the case of lump-sum taxation disappear.

Moreover, we can observe that a balanced-budget reduction of public debt carried out through an increase in the consumption tax rate spurs capital accumulation and growth as the change in public debt redistributes wealth among generations, while consumption taxation is *per se* neutral. Under this tax experiment the economy is subject to transitional dynamics. The short-run effects of consumption taxation on real growth and private consumption to national income ratio are smaller than the long-run effects.

Finally, revenue-neutral proposals that aim at reducing output taxes in favour of expenditure taxes are successful in terms of growth stimuli, because the reduction of the distortions of the output tax, that works in the growth-enhancing direction, is accompanied by a neutral change of the consumption tax.

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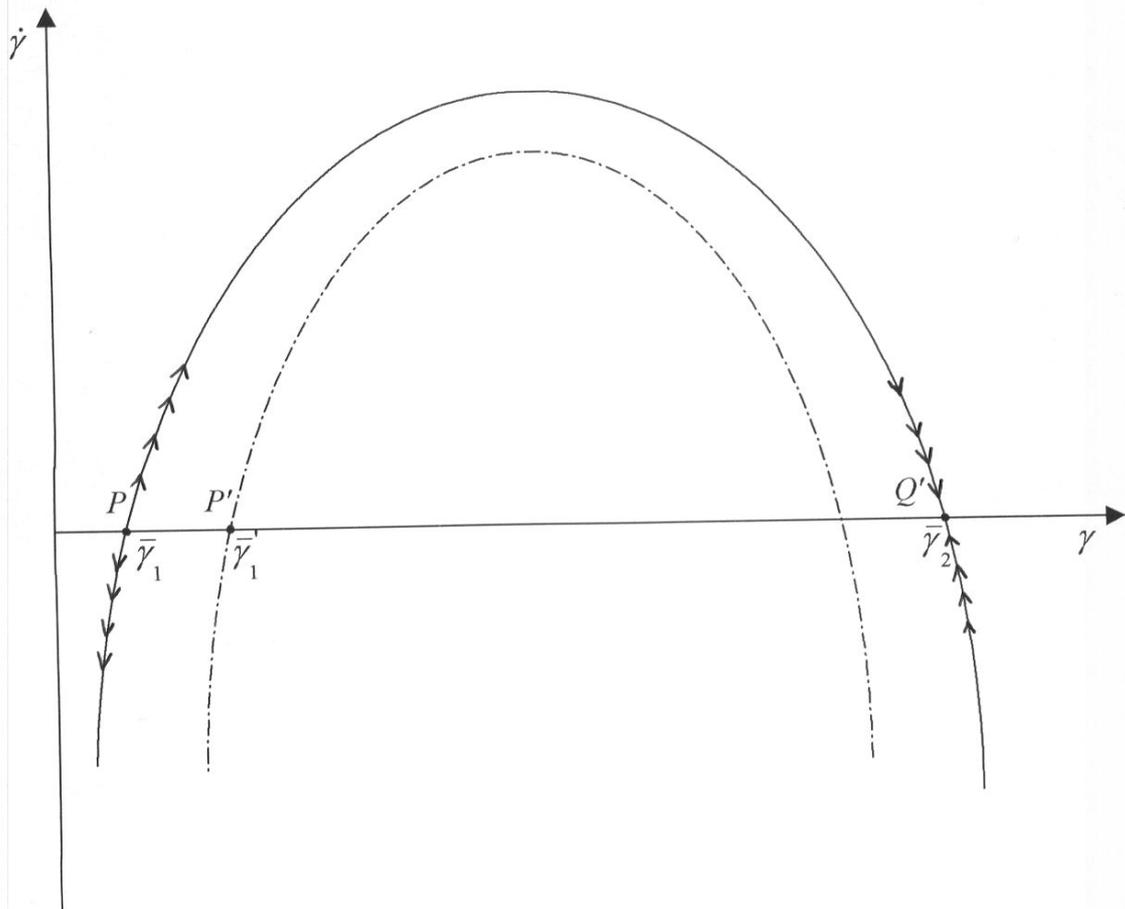


Figure 1  
 Effect of an increase in  $\tau_c$  when tax revenues are lump-sum rebated to consumers

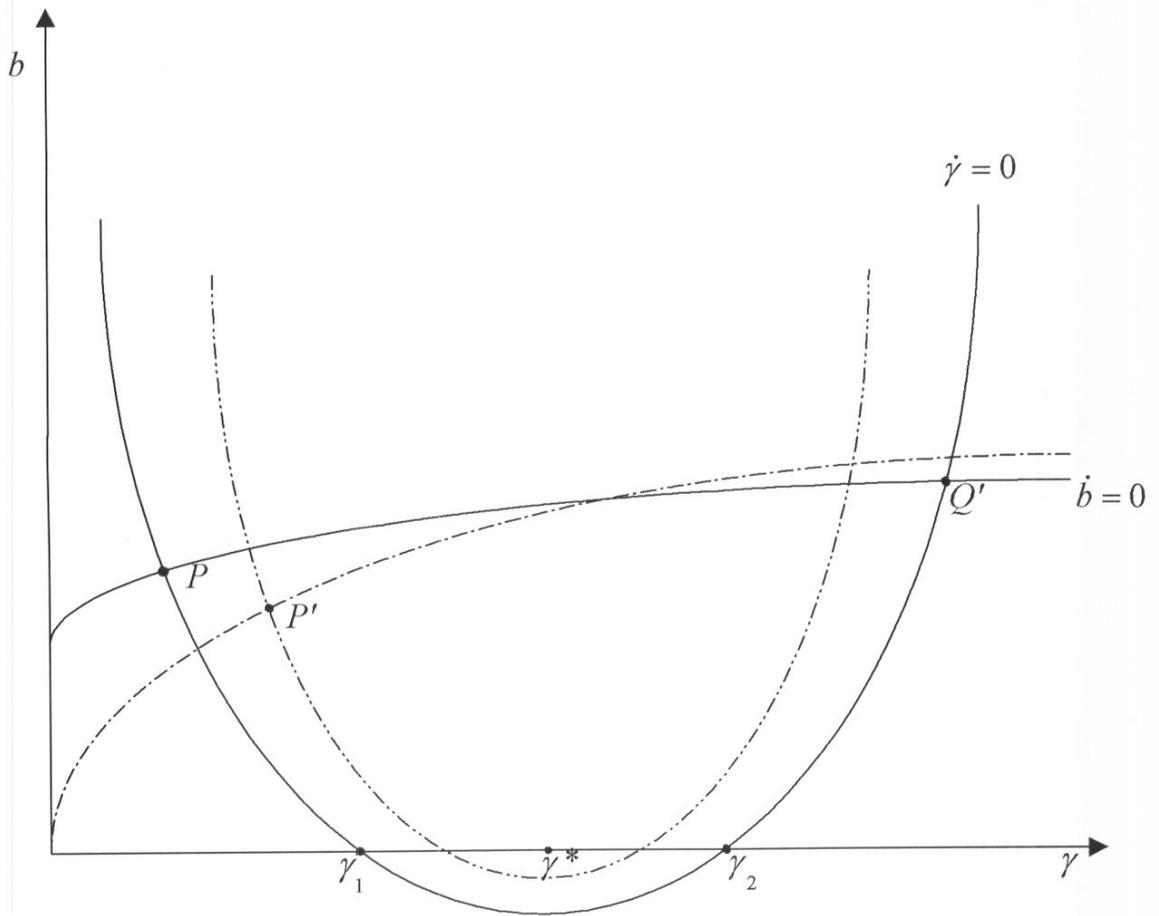


Figure 2  
Effects of an increase in  $\tau_c$  accompanied by a compensatory adjustment of  $b$

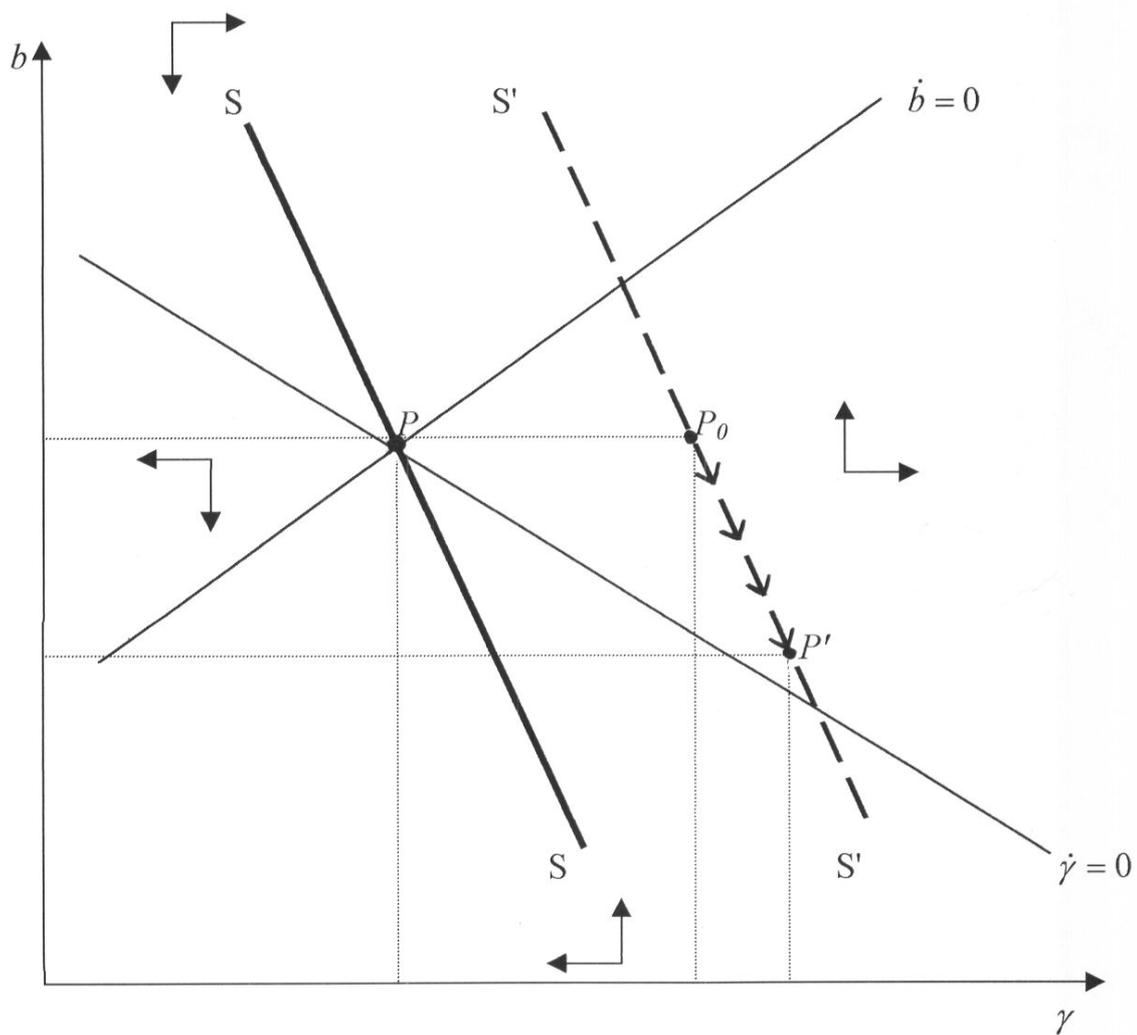


Figure 3  
 Dynamic effects of an increase in  $\tau_c$  accompanied by  
 a compensatory adjustment of  $b$